

VOTE-BUYING AND GROWTH

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Vote-buying is widely used by parties in developing countries to influence the outcome of elections. We examine the impact of vote-buying on growth. We consider a model with a poverty trap where redistribution can promote growth. We show that vote-buying contributes to the persistence of poverty as taxed wealthy people buy votes from poor people. We then show that there exists a democratic constitution that breaks vote-buying and promotes growth. Such a constitution involves rotating agenda setting, a taxpayer-protection rule, and repeated voting. The latter rule makes vote-buying prohibitively costly.

Keywords: Vote-Buying, Political Economy, Poverty Traps, Economic Development, Voting Rules, Repeated Voting

1. INTRODUCTION

Various studies suggest that vote-buying is an instrument widely used by parties in developing countries to influence the outcome of elections.¹ For example, buying votes is a long tradition in Mexico, the Philippines, Senegal, Taiwan, and Thailand. In the 2002 (community-level) elections in the Philippines, an estimated 3 million people were offered some form of payment. This corresponds to about 7% of all adults allowed to vote. In Thailand, 30% of the heads of households surveyed in a national sample said that they had been offered money during the 1996 general election. In Taiwan's 1999 election, 27% of a random sample of voters reported that they had accepted cash offers during previous electoral campaigns.²

However, if vote-buying occurs, then the success of redistribution policies used to overcome poverty may be endangered. Vote-buying may be bad for society and may in particular prevent growth-promoting redistribution policies. There is both theoretical and empirical evidence supporting this view. Buchanan and Tullock (1962) argue that in vote markets, minority groups—for example, the

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poor—are likely to face higher transaction costs than others, and may therefore become victims of income redistribution. Aghion and Bolton (2003) formalize the fear expressed by Schelling (1960) that vote-trading tends to increase the scope of the expropriation of voters. Barro (2000) and Docquier and Tarbalouti (2001) analyze the potential effects of vote-buying on redistribution in developing countries, with special reference to economic growth. They argue that some (rich) groups may have an incentive to buy votes in order to prevent redistribution. Their main result is that vote-buying is likely to reduce growth-promoting redistribution.

The objective of this paper is twofold. First, we analyze whether vote-buying can explain why democratic societies in many developing countries have been caught in a poverty trap. This is a positive analysis. We use a simple political economy model in which societies vote on growth-promoting redistribution, and combine it with the vote-buying model developed by Groseclose and Snyder (1996).³ We show that growth-promoting redistribution is impossible, as people burdened by taxes buy votes of poor people, and consequently, poverty persists.

Second, as a normative analysis, we suggest a set of constitutional rules that enable a society to break the negative consequences of vote-buying. Such rules must balance three requirements: proposals for growth-promoting redistributions must be made, such proposals must be approved by a majority, and rich people must be protected from excessive taxation, as well as from the threat of becoming poor. A democratic constitution that fulfills these requirements and thus promotes growth is called a growth-promoting constitution. Our main result is that such a growth-promoting constitution exists. It consists of a repeated-voting rule, a rotating agenda-setting rule, and a taxpayer-protection rule.

Repeated voting on the same proposal helps a society to break the negative consequences of vote-buying. The main intuition runs as follows: Under repeated voting, a proposal that has been rejected will be brought to vote again. This procedure can be repeated a fixed number of times. Once the proposal is accepted, the process ends immediately. Such a repetition of the voting on a single proposal makes vote-buying prohibitively costly, as the buyers of votes have to pay for votes in each period.

In order to promote growth, the repeated-voting rule will be combined with rotating agenda-setting, ensuring that growth-promoting redistribution proposals are made, and with a taxpayer-protection rule guaranteeing that richer people do not slide back into poverty.

The paper is organized as follows: In the next section we outline the related literature. The basic model is presented in Section 3. In Section 4 we discuss the policy necessary to overcome a poverty trap. In Section 5 we present the vote-buying model and we outline the political framework. In Section 6 we show that if vote-buying is possible, overcoming a poverty trap is not possible. In Section 7 we introduce repeated voting and show that a growth-promoting constitution exists under vote-buying. Section 8 concludes.

2. REVIEW OF THE LITERATURE

This paper is related to two different strands of the literature. First, there is a large literature dealing with the existence and persistence of poverty traps. Our focus on human capital and redistribution in a model with a poverty trap starts from the seminal contribution of Galor and Zeira (1993) [see also the important contribution of Azariadis (1996) and the survey of Azariadis and Stachurski (2005)]. Additionally, poverty traps are often connected with child labor, because poverty often means that children have to work to supplement a family's income. There is also a large literature on this subject. For an overview, see Jafarey and Lahiri (2001) and Basu and Tzannatos (2003).

Second, this paper refers to the constructive constitutional economics approach, which goes back to Buchanan and Tullock (1962). This approach deals with the design of new constitutional rules that might be helpful in democratic decision-making. Recent papers on constitutional design focus on optimal majority rules in the context of reforms and public goods provision.⁴ In this paper, we examine how democratic rules, such as a taxpayer-protection rule and a repeated-voting rule, can help to ensure that proposals for growth-promoting redistributions are made, such proposals are approved by a majority, and rich people are protected from excessive taxation and from the threat of becoming poor. In the concluding section we comment on how such constitutions might be implemented.

3. THE BASIC MODEL

3.1. Output Production and Human Capital Formation

We consider an overlapping-generations (OLG) model in which individuals live for two periods and where human capital accumulation is a major source of economic growth. The periods are labeled childhood and adulthood, respectively. For simplicity, we assume that each household comprises one adult and one child. We consider a society $\Omega = \{1, \dots, n\}$ consisting of $n > 3$ households, where n is assumed to be odd.⁵ A generic household is indexed by i . In the basic model, all households are alike and we drop the index.

We now turn to output production and consider an aggregate consumption good. For simplicity, let us assume that the human capital of adults is the only input factor needed for production. There are constant returns to scale, and all output will accrue to the households as income. We use $\lambda_t \in [1, \infty)$ to denote the human capital of an adult in period t . The condition $\lambda = 1$ for the society as a whole can be thought of as a state of backwardness. The level of output in period t produced by an adult who has a human capital endowment of λ_t is given by

$$y_t = \alpha \lambda_t, \quad (1)$$

where $\alpha \in (0, \infty)$ denotes the marginal productivity of human capital.

We now turn to the formation of human capital. We assume that in period t adults can make educational investments; i.e., they can use part of their income

to invest in the human capital of their children. We use $e_t \in [0, \infty]$ to denote the educational investments of an adult in period t . These costs can be interpreted in different ways. For instance, they may be the direct costs of school attendance. If school attendance is free of charge, they may represent foregone income, as schooling may reduce the time children can contribute to household production. The child's human capital endowment when reaching adulthood at time $t + 1$ is given by

$$\lambda_{t+1} = h(e_t) + 1. \quad (2)$$

The function $h(\cdot)$ represents the human capital technology. $h(\cdot)$ is assumed to be a continuous, strictly increasing, and differentiable function in e_t , where $h(0) = 0$, i.e., no investments in education, which leads to a human capital level amounting to 1. Equation (2) implies that educational investments are necessary for the formation of human capital in the next generation, i.e., for $\lambda_{t+1} > 1$.

3.2. The Household's Behavior

We assume that all allocative decisions lie in the adult's hands. We rule out any bequests and the possibility of debts, so that (1) is the current real income, used entirely for consumption,⁶ denoted by c_t , and educational investments e_t . The family's budget constraint is given by

$$c_t + e_t \leq y_t. \quad (3)$$

Adults are assumed to be altruistic; i.e., they want to maximize current consumption and educational investments for their children. Let an adult's preference ordering be representable by the continuous, strictly increasing, differentiable, strictly quasi-concave function $u(c_t, e_t)$ ⁷ and consider the problem

$$\max_{c_t, e_t} \{u(c_t, e_t)\} \quad \text{subject to} \quad (4)$$

$$c_t + e_t \leq \alpha \lambda_t$$

$$e_t, c_t \geq 0.$$

In view of the assumptions on $u(\cdot)$, this problem has a unique solution, denoted by $(c^o(\alpha \lambda_t), e^o(\alpha \lambda_t))$, which is continuous in λ_t .

We make the following two assumptions regarding the optimal choices of $(c^o(\alpha \lambda_t), e^o(\alpha \lambda_t))$:

Altruism is only operative if the human capital of adults, or equivalently their income y_t , is sufficiently large. Therefore, we assume that there exists a critical value $\lambda^S > 1$ such that

$$\begin{aligned} e^o(\alpha \lambda_t) &= 0 \quad \forall \lambda_t \leq \lambda^S, \\ e^o(\alpha \lambda_t) &> 0 \quad \forall \lambda_t > \lambda^S. \end{aligned} \quad (5)$$

Both goods are noninferior, i.e.,

$$\begin{aligned}\frac{\partial c^o(\alpha\lambda_t)}{\partial \lambda_t} &> 0 \quad \forall \lambda_t \geq 1, \\ \frac{\partial e^o(\alpha\lambda_t)}{\partial \lambda_t} &> 0 \quad \forall \lambda_t > \lambda^S.\end{aligned}\tag{6}$$

A typical example that satisfies both assumptions is Stone–Geary preferences, which are widely used in development economics [see, e.g., Basu and Van (1998) and Bell and Gersbach (2009)]. These preferences are given by

$$u(c_t, e_t) = \begin{cases} (c_t - c^S)e_t & \text{if } c_t \geq c^S \\ c_t - c^S & \text{otherwise,} \end{cases}$$

where c^S is the critical consumption level above which adults are motivated to invest in schooling. Hence, $\lambda^S = c^S/\alpha$. It is readily verified that condition (6) holds.

3.3. Dynamics

Returning to (2) in the light of (5), we obtain

$$\lambda_{t+1} = \begin{cases} 1 & \forall \lambda_t \leq \lambda^S \\ h(e^o(\alpha\lambda_t)) + 1 & \forall \lambda_t > \lambda^S. \end{cases}\tag{7}$$

In view of the assumption that $\lambda^S > 1$, it follows from the first part of (7) that the state of backwardness ($\lambda = 1$ for the society as a whole) is a locally stable steady state. Henceforth, we will refer to this steady state as the poverty trap.

To describe the dynamics of (7) for all $\lambda_t > \lambda^S$, we have to consider the derivative

$$\frac{d\lambda_{t+1}}{d\lambda_t} = \frac{\partial h(e^o(\alpha\lambda_t))}{\partial e^o(\alpha\lambda_t)} \cdot \frac{\partial e^o(\alpha\lambda_t)}{\partial \lambda_t},\tag{8}$$

which is strictly positive, as $\partial h(e^o(\alpha\lambda_t))/\partial e^o(\alpha\lambda_t) > 0$ and $\partial e^o(\alpha\lambda_t)/\partial \lambda_t > 0$ for all $\lambda_t > \lambda^S$.

In the following, we consider the case where the human capital technology is sufficiently productive, i.e.,

$$\frac{\partial h(e^o(\alpha\lambda_t))}{\partial e^o(\alpha\lambda_t)} \cdot \frac{\partial e^o(\alpha\lambda_t)}{\partial \lambda_t} > 1$$

for all $\lambda_t > \lambda^S$. In this case, there exists a second threshold λ^* ($\lambda^* > \lambda^S$), which is given as follows:⁸

$$\lambda_{t+1} = \lambda^* = h(e^o(\alpha\lambda^*)) + 1.$$

λ^* is a second stationary level of human capital, where adults and their offspring share the same level of human capital. In the following we use $y^* = \alpha\lambda^*$ to denote

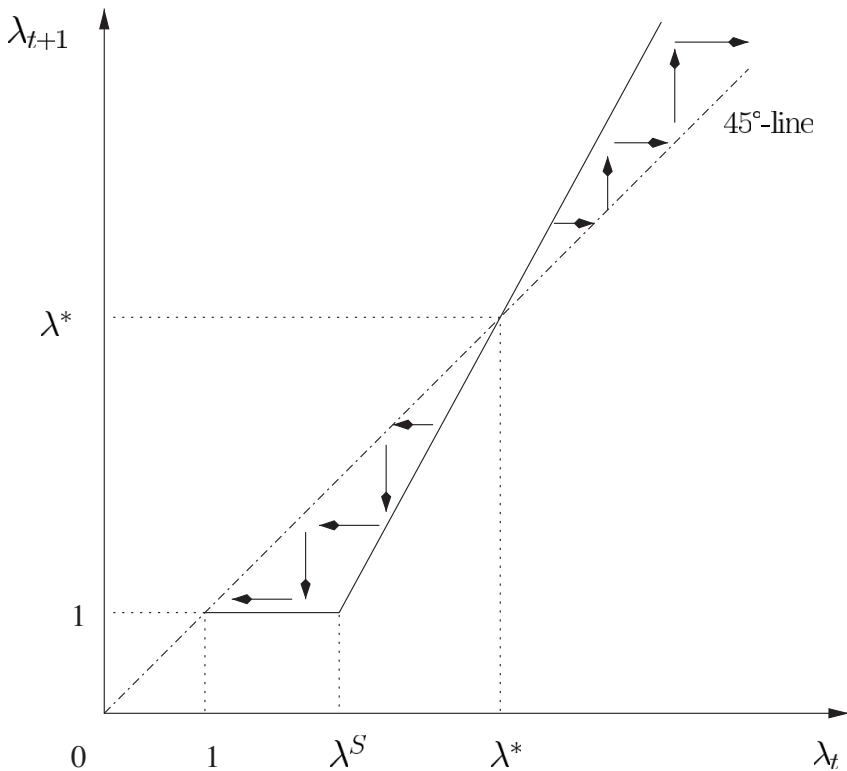


FIGURE 1. Human capital formation.

the output of an individual with a human capital level of λ^* . Note that λ^* is an unstable steady state. The dynamic of our model is shown in Figure 1.

Figure 1 illustrates how long-term growth depends on the size of educational investments made by the adults, which, in turn, depends on their human capital level. For example, if the educational investments $e^o(\alpha\lambda_t)$ of the adults in period t are not sufficiently large, i.e., $e^o(\alpha\lambda_t) < e^o(\alpha\lambda^*)$, then the human capital of these children and their offspring will be smaller than λ^* in subsequent periods, and subsequent generations will fall back into the poverty trap. However, if the adults choose $e^o(\alpha\lambda_t) > e^o(\alpha\lambda^*)$, then the human capital of these adults' descendants in the subsequent periods will be greater than λ^* and human capital will grow in subsequent periods (see Figure 1).

In short, overcoming the poverty trap requires that uneducated individuals be given sufficient support for the adults to be able to choose $e^o(\alpha\lambda_t) > e^o(\alpha\lambda^*)$. In the following, we will call an individual educated if his human capital is larger than λ^* , i.e., if he will afford schooling for his offspring that yields increasing human capital and output. Moreover, we will call a society educated if all its members have human capital larger than λ^* .

It is important to stress that growth-promoting redistribution is optimal for the society from a utilitarian perspective, taking all generations into account, if future generations have a sufficiently high weight, i.e., the discount factor is not too low. This justification rests on the following externality: The improvements in all future generations' welfare that stem from better education of today's children are not fully reflected in the preferences of today's parents. This holds because parents care about their children's education, but not about what happens subsequently. If, as arguable, the social planner has a longer time horizon than individual households, then the case for redistribution to promote schooling is, in principle, established [cf. Bell and Gersbach (2009)].

4. REDISTRIBUTION TO OVERCOME POVERTY

In the following, we assume that the whole society is initially ($t = 0$) in a state of backwardness; i.e.,

$$\lambda_0^i = 1 \quad \forall i \in \Omega,$$

which represents a worst-case scenario.⁹ The broad policy objective is to educate the whole society in order to enable all its members to escape from this backwardness; i.e.,

$$\lambda^i > \lambda^* \quad \forall i \in \Omega.$$

The instruments for this purpose are taxation and subsidization. Let τ_t^i denote the tax levied on the income of household i in period t . At the beginning of each period t , some individuals will be subsidized from the ensuing tax revenue. We use s_t^i to denote the subsidy each household i will receive in period t . We suppose that households are not simultaneously taxed and subsidized. Because households in a state of backwardness have few resources, we assume that there is a subsistence level c^{sub} for an adult-child household that must be guaranteed under all circumstances. The taxation of a household i caught in the poverty trap is therefore assumed to be constrained by

$$\alpha - \tau_t^i \geq c^{\text{sub}},$$

where α is the income of households with $\lambda_t = 1$. In particular, the tax must fulfill the following condition:

$$\tau_t^i \leq \alpha - c^{\text{sub}} =: \tau^{\text{sub}}.$$

Next, we assume that $\tau^{\text{sub}} > 0$. It is plausible for τ^{sub} to be small, as households caught in the poverty trap may already be close to the subsistence level c^{sub} .

We define s^* as the subsidy a household that is in a state of backwardness needs to achieve a human capital level of λ^* in the subsequent period. Hence, s^* is defined by the implicit equation

$$h(e^o(\alpha + s^*)) + 1 = \lambda^*.$$

We note that s^* is uniquely defined. As $\lambda^* = h(e^0(\alpha\lambda^*)) + 1$ we note that the subsidy s^* has to fulfill

$$h(e^0(\alpha + s^*)) = h(e^0(\alpha\lambda^*)). \quad (9)$$

Hence, $s > y^* - \alpha = \alpha(\lambda^* - 1)$, as the uneducated must be subsidized by an amount larger than the income differences between $\alpha\lambda^*$ and α .

To overcome the poverty trap permanently, uneducated individuals have to be given sufficient support for educational investments that yield increasing human capital. Accordingly, we define \bar{s} as the subsidy a household in a state of backwardness needs to achieve a human capital level larger than λ^* in the subsequent period. Hence, \bar{s} is given by the equation

$$\bar{s} = s^* + \epsilon,$$

where ϵ is small but positive.

We now look at households in a particular period t that have received subsidies \bar{s} in the previous period and thus, $\lambda_t^i > \lambda^*$. If taxation of such households is very high, education of the offspring will be low and their human capital may fall below λ^* . Such a slide back into poverty does not happen if

$$\alpha\lambda_t^i - \tau_t^i \geq \alpha + \bar{s},$$

which defines an upper level for the taxes of educated households, denoted by τ^* :

$$\tau^* := \alpha(\lambda_t^i - 1) - \bar{s}.$$

The total government revenues in period t are denoted by B_t . The budget constraint in a period t is given by

$$B_t = \sum_{i=1}^n \tau_t^i \geq \sum_{i=1}^n s_t^i.$$

Throughout the paper, we assume that

$$\frac{n-1}{2} \tau^{\text{sub}} \geq \bar{s}.$$

That is, the taxation of $(n-1)/2$ uneducated households is sufficient to subsidize at least one uneducated household with \bar{s} .

5. THE VOTE-BUYING GAME

5.1. The Game Form

In the following, we consider the case where individuals who will be taxed if the proposal is implemented (henceforth called *taxpayers*) may engage in up-front vote-buying. Up-front vote-buying is a binding agreement that gives an individual full control of the vote of another individual in exchange for an up-front payment.

The central idea is that taxpayers form a coalition to defend themselves against the adoption of tax/subsidy proposals through vote-buying. As an illustration, consider the following real-world example. In Mexico, vote-buying was used by the ruling party and its supporters to influence the outcome of elections. Governors from Mexico's ruling party, the Institutional Revolutionary Party (PRI), misused the poverty-alleviation program to buy the votes of poor voters and win the elections in 2000, and to prevent the extensive redistribution proposed by the opposition.¹⁰

We assume that vote-buying is legally forbidden, but the agenda-setter cannot observe which individuals are purchased and which are not. This implies that vote-buyers and -sellers face no risk of punishment. Because the agenda-setter is aware of vote-buying, he may have an incentive to make a proposal that includes subsidy payments to untaxed individuals, to make vote-buying expensive.

For simplicity, we assume that each taxpayer in the coalition formed to prevent the adoption of a redistribution proposal will have the same bargaining power; i.e., if the taxpayers form a coalition to engage in vote-buying, each taxpayer has to pay the same amount. Moreover, we assume that the coalition of taxpayers can monitor the casting of votes by the purchased individuals and can prevent deviations. In reality, there are several strategies for the vote-buyers to generate and enforce compliance.¹¹ For example, vote-buyers can instruct voters to fold the ballot in a distinctive way, or to put a pinhole in one corner of the ballot, so that vote-buyers can easily verify whether the voters have voted as instructed. Another way is to give a voter a fake or stolen pre-marked ballot before entering the polling station. The voter casts the filled-in ballot and gives the official blank ballot to another voter waiting outside. This voter fills out the (received) ballot to the buyer's satisfaction, and goes back into the polling station and repeats the process. Another common practice is to pay voters to abstain from voting, thereby preventing them from casting ballots for the opponent.

We apply the vote-buying game developed by Groseclose and Snyder (1996). We consider a sequential game with the agenda-setter moving first and the coalition of taxpayers moving last. This assumption could be justified by the observation that the payments for votes by the agenda-setter are part of his proposal and if it is costly to change proposals—which we will assume in the following—then the coalition of taxpayers is indeed able to move last.

We now turn to the sequence of the vote-buying game. The timing of events in period t can be summarized as follows:

- (1) An individual is randomly chosen to set the agenda. The agenda-setter either announces a redistribution proposal or makes no proposal.
- (2) If the agenda-setter announces a proposal, then
 - (i) the taxpayers can form a coalition and decide on the basis of this proposal whether or not to buy votes using their personal wealth;
 - (ii) vote-buying does or does not take place;
 - (iii) the society holds a vote on the implementation of the proposal;

- (iv) the proposal is adopted if it receives more votes than required by some voting rule.

The status quo will prevail if the agenda-setter makes no proposal or if a redistribution proposal is not adopted. We stress that the agenda-setter uses government coercion to redistribute money by taxing a share of citizens and subsidizing votes: He tries to get support from citizens by making proposals that are sufficiently attractive to a majority of voters. The agenda-setter does not use his private wealth. A coalition of taxpayers, in contrast, use their private wealth to buy votes and to defeat the proposal.

In the vote-buying stage, individuals know who will be taxed and who will receive subsidies if a proposal is accepted. In particular, at this point in time, the coalition of taxpayers is perfectly informed about the offers made by the agenda-setter.

Several remarks about the broader context of our vote-buying game are in order. First, the vote-buying setup in this section is an example of democracy working under weak institutions, as we assume that neither vote-buyers nor vote-sellers face a risk of punishment. However, we assume that taxpayers cannot form an antidemocratic elite to threaten democracy, as in Brender and Drazen (2009), and that the incumbent cannot use violence or coercion to repress groups of voters—and in particular swing voters—as in Robinson and Torvik (2009). Hence, the democracy in our paper is in the middle between two poles, strong democracies, in which vote-buying would be punished, and unconsolidated, fragile democracies.¹² This perspective is reinforced further by our assumption that the agenda-setter is chosen randomly. In essence, this assumption rules out the possibility that a single agenda-maker (e.g., the incumbent government) can always control the agenda and can ensure via vote-buying through benefits, jobs, or protection that only proposals that benefit the incumbent are made and adopted. Hence, the assumption that the agenda-setter is chosen randomly reflects the fact that there are several powerful democratic groups in the society, which deter each other from monopolizing the agenda power forever.

There is a further justification for the agenda-selection rule. Random selection is a method for choosing a proposal-maker fulfilling the democratic requirement that each person should have the same chance to make a proposal (anonymity principle). It is widely used in the literature by political scientists and economists to model democratic agenda-setting [Mueller et al. (1972), Baron and Ferejohn (1989); Frey and Stutzer (2006)] and is actually used in India, for instance [Duflo (2005), p. 669].

5.2. Constitutional Rules

In this section, we explore the capacity of democratic constitutions to promote growth. Such a constitution is a set of rules that specify how the agenda-setter is chosen and how decisions are made. In order to give democracy a good chance to overcome poverty, we introduce the following set of rules:¹³

The democratic agenda-setting process is specified as follows:

Rotating agenda setting (RoA): The agenda-setter is selected randomly. In the first period, each individual i has the opportunity to make a proposal. In subsequent periods, only individuals who have not set the agenda in previous periods can apply for agenda-setting. Each individual i allowed to make a proposal has the same chance of setting the agenda.

This rule implies that the number of permitted reelections is zero. It ensures that each individual will be the agenda-setter at some point in time and will therefore have the chance to make an education-enhancing redistribution proposal on which the society will hold a vote.

Moreover, we assume that a proposal has to satisfy the following agenda rule:

Balanced budget (BB): A proposal has to satisfy a balanced budget; i.e.,

$$\sum_{i=1}^n \tau_t^i - \sum_{i=1}^n s_t^i = 0, \quad \forall t.$$

By a balanced budget being required in each period, the possibility of capital market-financed subsidies for education is excluded. Thus, we analyze a worst-case scenario in the following. Obviously, a society that can be educated without access to capital markets can also be educated if it has access to them.

As a decision rule, we use a variant of the flexible-majority rules [see, e.g., Gersbach (2004, 2011)] in order to limit the taxation of educated households so that they do not fall back into poverty. For this purpose, we denote by $\bar{\tau} > 0$ an arbitrary tax level that serves as a threshold value in the definition. We define

$$\tau_t^{\max} = \max_{i \in \Omega} \tau_t^i.$$

Threshold flexible majority rule (TFM $[\tau_t^{\max}, \bar{\tau}]$): Under this rule, the share of votes needed to implement a proposal, denoted by $m(\tau_t^{\max}, \bar{\tau})$, jumps from 1/2 (simple majority) to 1 (unanimity) if any individual i is taxed higher than the threshold tax $\bar{\tau}$ stated in the constitution:

$$m(\tau_t^{\max}, \bar{\tau}) = \begin{cases} \frac{1}{2} & \text{if } \tau_t^{\max} \leq \bar{\tau}; \\ 1 & \text{if } \tau_t^{\max} > \bar{\tau}. \end{cases}$$

The flexible-majority rule effectively operates as a tax protection rule. It ensures that a winning majority for the proposal can be obtained if and only if educated adults are not taxed adversely, i.e., if $\tau_t^{\max} \leq \bar{\tau}$. As soon as an agenda-setter suggests an adverse tax scheme, i.e., $\tau_t^{\max} > \bar{\tau}$, the constitution requires unanimous agreement, which, de facto, makes expropriation impossible to implement. Hence, if a household has income $\alpha\lambda_t$, de facto it will be protected against expropriation of income $\alpha\lambda_t - \bar{\tau}$.

We note that the threshold flexible-majority rule is an extreme form of protection, as unanimity is required to tax someone over a constitutional threshold. Our results would work with weaker forms of threshold flexible-majority rules, as long as richer people are protected from complete income dispossession. For

instance, one could have a three-quarters quorum requirement for the acceptance of proposals involving taxation higher than $\bar{\tau}$, coupled with tax protection such that rich people will be able to retain income $\alpha\lambda^*$. In such cases, human capital in a lineage may decline during particular periods and tax rates for particular households may become very high. Nevertheless, after some time, all individuals will be educated and human capital will not fall below λ^* . However, the formal analysis of this more flexible scheme becomes much more involved.

A particular form for m of such threshold flexible-majority rules is to set $\bar{\tau} = \min\{\tau^{\text{sub}}, \tau^*\}$. Recall that τ^{sub} is the highest taxation allowed for households in a state of backwardness, whereas τ^* is the highest tax burden for an already subsidized household that does not endanger educational investments in the future. Hence, the minimum of τ^{sub} and τ^* ensures that uneducated households will not fall below the subsistence level, and educated households will not fall back into the poverty trap.

5.3. Equilibrium Concept

Given the constitutional rules described in Section 5.2, we will look at subgame-perfect equilibria in the vote-buying game. It is convenient to introduce the following tie-breaking rule for agenda-setting. We assume

TR 1: Individual i will apply for agenda-setting if and only if he can strictly improve his utility by agenda-setting.

That is, the agenda-setter expects that he can make a proposal with $s_t^{\text{ag}} > 0$ that will be adopted, where s_t^{ag} denotes the subsidy of the agenda setter. Alternatively, we can assume that there are small but positive fixed costs for agenda-setting.

5.4. Voting Behavior of Unbribed Individuals

In this section, we examine the voting behavior of unbribed individuals. Recall that we have assumed that a proposal either levies taxes on individuals (including a zero tax rate) or provides subsidies. Obviously, taxpayers will vote against the proposal, whereas subsidized individuals who have not been bribed will support it. If an unbribed individual i is neither taxed nor subsidized, then he is indifferent between supporting and rejecting the proposal. As a tie-breaking rule, we assume

TR 2: An unbribed individual i will support the proposal if

$$s_t^i = \tau_t^i = 0.$$

5.5. Incentives to Buy and Sell Votes

In the next step, we examine the incentives to buy and sell votes. The incentive of the coalition of taxpayers to buy votes depends on whether the agenda-setter proposes an adverse tax scheme or not. If the tax scheme satisfies $\tau_t^{\text{max}} > \bar{\tau}$,

then unanimity rule prevails. In this case, there is no need for the taxpayers to engage in vote-buying, as each individual has the power to vote a proposal down.

If the agenda-setter suggests a tax scheme with $\tau_t^{\max} \leq \bar{\tau}$, the simple majority rule prevails. Given this situation, the agenda-setter and the coalition of taxpayers will be interested in obtaining a majority of votes for and against the proposal, respectively, while spending as little as possible. That is, they will compete for the votes of the individuals who will not be taxed if the proposal is accepted. We now turn to the payment promises made by the agenda-setter and by the coalition of taxpayers to the untaxed individuals. We define

$$\text{NT} = \{i \in \Omega \mid \tau_t^i = 0 \wedge i \neq \text{ag}\}$$

as the set of untaxed individuals in which the agenda-setter is not included. Let s_t^i denote the offer from the agenda-setter to the untaxed individual $i \in \text{NT}$, and let p_t^i denote the payment offer of the coalition of taxpayers to the untaxed individual $i \in \text{NT}$.

Both the agenda-setter and the coalition of taxpayers have an incentive to bribe untaxed individuals if and only if the expected tax revenues B_t are at least as high as their total payment promises to the untaxed individuals. If this is not the case, vote-buying will not occur. Alternatively, B_t can be interpreted as the budget or the willingness to pay for implementing and preventing the proposal on the part of the agenda-setter and the coalition of taxpayers, respectively. The agenda-setter uses subsidies in his proposal that work as bribes. The coalition of taxpayers use their private wealth to buy votes.

The incentives of these individuals can be formalized as follows:

VB(1): The agenda-setter will buy votes if and only if

$$B_t \geq \sum_{i \in \text{NT}} s_t^i, \quad \text{with } s_t^i \geq 0.$$

VB(2): The coalition of taxpayers will buy votes if and only if

$$B_t \geq \sum_{i \in \text{NT}} p_t^i \quad \text{with } p_t^i \geq 0.$$

The preference of both the agenda-setter and the coalition of taxpayers is to win at minimal cost. In equilibrium, the agenda-setter's winning utility is

$$s_t^{\text{ag}} = B_t - \sum_{i \in \text{NT}} s_t^i \geq 0$$

and his losing utility is zero, where s_t^{ag} is the subsidy for the agenda-setter and $\sum_{i \in \text{NT}} s_t^i$ is the total of all payments incurred by the agenda-setter (including zero subsidies to some of the untaxed individuals). In contrast, in equilibrium, the utility from winning for the coalition of taxpayers amounts to $-\sum_{i \in \text{NT}} p_t^i$ and its utility from losing is $-B_t$, where $\sum_{i \in \text{NT}} p_t^i$ is the total of all payments incurred by

the coalition (including zero payment offers to some of the untaxed individuals) and

$$B_t - \sum_{i \in \text{NT}} p_t^i \geq 0$$

is the value from winning if the majority votes against the proposal in equilibrium. Note that the coalition of taxpayers will only engage in vote-buying if it knows that it will win in equilibrium. Otherwise, the coalition will not buy any votes, as the money would be wasted.

We now regard the untaxed individuals, who may have an incentive to sell their votes to the coalition of taxpayers. As vote-buying is illegal, we assume that there are positive moral costs of vote-selling, denoted by ϕ per vote.¹⁴ A bribed individual i will support the proposal if $s_t^i + \phi > p_t^i$ and reject it if $s_t^i + \phi < p_t^i$. If $s_t^i + \phi = p_t^i$, then the bribed individual i will be indifferent between supporting and rejecting the proposal. As a tie-breaking rule, we assume

TR 3: A bribed individual i will sell his vote to the coalition of taxpayers if

$$s_t^i + \phi = p_t^i.$$

6. THE IMPOSSIBILITY RESULT

In this section, we examine the outcome of the entire game with constitutional rules as set out in Section 5.2 and the tie-breaking rules TR 1–TR 3. We use T to denote the number of periods a democratic society needs to educate itself. Recall our assumption that initially ($t = 0$), the whole society is in a state of backwardness, i.e., $\lambda_0^i = 1 \forall i \in \Omega$. If vote-buying is possible, we obtain the following result:

PROPOSITION 1. *Consider the case of a democracy with a constitution that provides for*

rotating agenda-setting (RoA)¹⁵

threshold flexible majority rule (TFM $[\tau_t^{\max}, \bar{\tau}]$), with $\bar{\tau} = \min\{\tau^{\text{sub}}, \tau^\}$*

balanced budget (BB).

Such a democracy cannot educate a society in finite time; i.e., $T = \infty$, if vote-buying is possible and if the moral costs of vote-selling are sufficiently small, that is, if

$$\phi < \frac{(n-3)\tau^{\text{sub}}}{(n-1)(n-2)}.$$

The proof is given in the Appendix.

The reason for the result of Proposition 1 is the following: Suppose that the randomly chosen agenda-setter in $t = 0$ makes a proposal where he taxes at

most $(n - 1)/2$ individuals with τ^{sub} to subsidize himself and other untaxed individuals. As the agenda-setter knows that the taxpayers can form a coalition after he has announced a proposal and buy the “cheapest” untaxed individuals of his proposal, the best thing he can do is to make a proposal with equal subsidies to all untaxed individuals. In contrast, the coalition of taxpayers has the advantage of buying only a small number of untaxed individuals to form a simple majority against the proposal, as all taxpayers will vote against the proposal. Moreover, it is sufficient to offer these individuals slightly more than the subsidies of the agenda-setter and the moral costs of vote-selling together to win their votes. If the moral costs of vote-selling are not too high, then vote-buying is always profitable for the coalition of taxpayers. This, in turn, implies that no proposal made by the agenda-setter will ever be accepted, because a majority will always vote against it. Because the agenda-setter expects that he cannot strictly improve his utility by agenda-setting, he will refuse to make a proposal. The preceding argument holds true for every period t . Hence, the education of a society is not possible in finite time, and the economy remains in a state of backwardness.

We note that for high moral costs of vote-selling, which effectively make vote-buying extremely costly and thus unprofitable for taxpayers, the constitution in Proposition 1 will induce education of the society in finite time. Rotating agenda-setting ensures that every part of the society will be subsidized by \bar{s} at least once, which will promote education and growth. The threshold flexible-majority rule guarantees that educated rich people are not excessively taxed and become poor. The result of Proposition 1 also applies to societies where a share of individuals is already educated.

COROLLARY 1. *Consider a society where some individuals are already educated, i.e., $\lambda_0 > \lambda^*$ holds for these individuals. A constitution with RoA, BB, and TFM cannot educate such a society in finite time if the moral costs of vote-selling are sufficiently small.*

The proof of this statement follows the same logic as the proof of Proposition 1 and is therefore omitted.

Corollary 1 states that the failure of the education of a society does not depend on the fact that the whole society is initially in the poverty trap. The reason for this result is the following: If some individuals are already rich and educated, the size of the expected tax burden may change. A change in the expected tax burden, however, affects the vote-buying budget of the agenda-setter and of the coalition of taxpayers in the same way. As the coalition of taxpayers will make its payment offers to the poor untaxed individuals after the agenda-setter has announced his proposal, the advantage of buying only a small number of untaxed individuals remains. Hence, the coalition of taxpayers is still able to bid in such a way that the proposal of the agenda-setter will be rejected, as long as the moral costs of vote-selling are sufficiently small.

7. REPEATED VOTING

7.1. The Rule

To eliminate the negative impact of vote-buying, we introduce repeated voting.¹⁶ The additional agenda rule is described as follows:

Repetition of Voting (RoV[R]): If the proposal of an agenda-setter i is rejected, the vote on that proposal will be repeated. A vote will be repeated R times. If the proposal is accepted, voting ends. However, if the proposal is rejected R times, the status quo prevails.

We now describe the sequence in period t in more detail. At the beginning of period t , the agenda-setter is allowed to make a proposal. In the next stage, the society holds a vote on the implementation of this proposal. If a majority vote in favor of the proposal, it is accepted. Otherwise, there will be a new vote. If the project is rejected again, then there will be another vote on this subject. This procedure will be repeated as long as the proposal is not accepted. However, repetition of voting stops if the proposal is rejected R times. In this case, the status quo will prevail.

7.2. The Main Result

With repeated voting, we can obtain a possibility theorem.

PROPOSITION 2. *Suppose vote-buying is possible. Consider the case of a democracy with a constitution that provides for*

*rotating agenda setting (RoA),
threshold flexible majority rule (TFM $[\tau_t^{\max}, \bar{\tau}]$) with $\bar{\tau} = \min\{\tau^{\text{sub}}, \tau^*\}$,
balanced budget (BB),
repetition of voting (RoV[R]),*

and the number of possible voting repetitions amounts to

$$R = \lceil R^* \rceil \quad \text{with} \quad R^* = \frac{(n-1)\tau^{\text{sub}}}{2\phi}, \quad \phi > 0.$$

Such a democracy can educate a society in finite time; i.e., $T < \infty$.

Note that $\lceil R^* \rceil$ denotes the minimal natural number larger than or equal to R^* . The proof of Proposition 2 is given in the Appendix.

The reason for the result of Proposition 2 is the following: Suppose that the randomly chosen agenda-setter in $t = 0$ makes a proposal where he taxes at most $(n-1)/2$ individuals with τ^{sub} to subsidize himself with \bar{s} . The possibility of repeated voting makes vote-buying prohibitively costly, as the coalition of taxpayers has to buy at least one untaxed individual in each vote to form a minimal coalition against the proposal. If the number of possible voting repetitions is sufficiently large, then vote-buying will not be profitable for the coalition of

taxpayers, because the total payments needed to prevent the implementation of the proposal will outweigh tax demand in the first round. Hence, it is optimal for the coalition of taxpayers not to engage in vote-buying in the first round. According to TR 2, all untaxed individuals will vote in favor of the proposal in the first vote, which implies that the proposal of the agenda-setter will be accepted. The rotating agenda-setting rule ensures that each poor individual will have the right to set the agenda in the future, which implies that all individuals will receive the required transfer \bar{s} . The threshold flexible-majority rule guarantees that educated rich people are not excessively taxed and become poor. Hence, a constitution consisting of RoV, RoA, and TFM promotes growth. As a result, the society will be educated in finite time.

7.3. Discussion

It is clear that RoV will only deter taxpayers from vote-buying if repeated voting actually reduces the wealth of the vote-buyer more than the acceptance of a proposal to support education for other people. This occurs if vote-buyers have to buy votes in each round. Thus, there always exists a number of voting repetitions such that accumulated payments of vote-buyers become large, even if the cost to buy votes in a single round is small. There are two additional considerations that impact on the required number of voting repetitions.

First, the organization of voting in itself is costly, as people have to be hired and voters need time to cast their votes. We have neglected these organizational costs so far. In practice, however, they have to be added, as they further decrease the wealth of the rich vote-buyers in each period and thus make voting repetition even more unattractive. Thus, organizational costs will decrease the number of vote repetitions necessary to prevent vote-buying.

Second, the success of a constitution consisting of RoV, RoA, and TFM might conceivably be endangered by “long-term vote-buying contracts.” A long-term vote-buying contract is a binding agreement that gives the vote-buyer full control of the vote of another individual for more than one vote, in exchange for an up-front payment. Such long-term contracts might allow vote-buyers to reduce the costs in a single round. However, long-term vote-buying contracts become more and more difficult, the more rounds there are for casting votes. The reason is that vote-buying contracts are illegal, and therefore they cannot be enforced by courts. Hence, attempts of vote-buyers to offer long-term contracts can be counteracted by increasing the number of possible voting repetitions.¹⁷

In our analysis, we have considered a society where the moral costs of vote-selling are relatively small, where vote-buyers and -sellers face no risk of punishment, and where vote-buyers can monitor the casting of the votes they bought perfectly. These features tend to apply to many developing countries. If we considered a society where, for example, the risk of being punished was very high or where the moral costs of vote-selling were large, vote-buying would be prohibitively costly and thus less attractive. This tends to hold for industrial countries.

8. CONCLUSIONS

This paper has provided two insights. First, we have shown that if agents can trade votes, and if the moral costs of vote-selling are not too large, the education of a democratic society is impossible. Hence, this society will remain in the poverty trap. This pessimistic result is due to the fact that the potential losers from redistribution have strong incentives to buy votes to prevent redistribution. This impossibility result may provide one possible explanation for why many developing countries have been caught in the poverty trap for such a long time.

Second, we have shown that a constitution consisting of a repeated voting rule, a rotating agenda-setting rule, and a threshold flexible-majority rule enables a society to escape the poverty trap if vote-buying is possible, as the opportunity of repeated voting makes vote-buying prohibitively costly, and therefore unattractive for the taxpayers. The threshold flexible-majority rule guarantees that rich people are not taxed excessively, which would impoverish them. Rotating agenda-setting ensures that each individual will have his turn in agenda-setting and will receive growth-promoting transfers.

Numerous issues deserve further scrutiny. For instance, it is important to look into the opportunities to introduce a growth-promoting constitution. Although, in principle, the standard “veil of ignorance” argument could be used, in overlapping-generation models, it may be sensible to use the requirement that the current generation of adults must support a new constitution. In such circumstances, delayed implementation could be used, which works as follows.¹⁸ Consider a proposal to introduce the growth-promoting constitution, coupled with the requirement that the constitution can only be abolished by a qualified majority. Moreover, suppose that if accepted, the constitution would become effective only after a delay—after the old generation had died. Then, as long as the current generation of adults is minimally and equally concerned¹⁹ about the well-being of its children and grandchildren, the current generation of adults will favor the proposal. The high-majority hurdle for its abolition would ensure that the rule will not be eliminated once it has been introduced.

NOTES

1. In the literature, different notions of “vote-buying” are discussed. For an overview see, for example, Schaffer (2006). In this paper, vote-buying is seen as a purely economic exchange where votes are traded for cash, for example.

2. See, for example, Hicken (2002), Rigger (2002), and Schaffer (2004).

3. The model has recently been generalized by Dekel et al. (2008), who allow a sequential and alternating bidding process over multiple rounds.

4. See, for example, Aghion and Bolton (2003), Aghion et al. (2004), and Gersbach (2004).

5. This assumption is not essential, but it simplifies our analysis, as it eliminates the possibility of a draw.

6. Consumption includes the consumption of the adult and of the child, which is often viewed as a fixed fraction of the adult’s consumption.

7. This formulation of altruism is convenient. An alternative way would be to use $u_t = v(c_t) + \beta u_{t+1}$ ($0 < \beta < 1$). At the cost of additional formal complexity, the same analysis can be performed for this way of expressing altruism.

8. The results remain unchanged if we assume the somewhat weaker conditions $d\lambda_{t+1}/d\lambda_t < 1$ for $\lambda_t = 1$ and $\lim_{\lambda_t \rightarrow \infty} d\lambda_{t+1}/d\lambda_t > 1$. This also guarantees the existence of the threshold λ^* . Our procedure can be applied to this setting.

9. Note that in reality, income distribution in developing countries is typically unequal. In the following sections, we show that our results also hold true if we assume that there initially exists a minority of “educated” rich households.

10. See, e.g., Pfeiffer (2004).

11. Schaffer (2006) gives a description of a number of strategies available to vote-buyers to generate and enforce compliance.

12. For a recent economic analysis of the way non-democratic societies evolve, see Acemoglu et al. (2009).

13. Indeed, Gersbach and Siemers (2005) show that without vote-buying the set of rules introduced in this section induce education-promoting redistribution and growth.

14. Alternatively, we can drop the assumption that the risk for vote-sellers of being arrested and punished is zero, and assume instead that there is a small, but positive probability that the agenda-setter can observe which individuals have been bought by the coalition of taxpayers. In this case, ϕ could also be interpreted as a risk premium demanded by the vote-sellers to compensate for the risk of being arrested and punished.

15. If there is a dynasty that holds the agenda-setting power, a society has a priori no possibility of overcoming the poverty trap, as is evident from the proof of Proposition 1.

16. We will use repeated voting to break the blockade against education-enhancing proposals induced by vote-buying. Repeated voting may also have other virtues. For example, Morton (1988) has shown that agents can acquire information on voter preferences by observing the results of early referenda and use that information in formulating a strategy for subsequent referenda. Repeated voting is actually applied in practice. For instance, the possibility of repeated referenda is allowed in the constitutions of the Republic of Tajikistan (1995) and of Slovakia (1992) (see, e.g., Article 31 of the Constitutional Law of the Republic of Tajikistan on a Referendum, or Article 99 of the Constitution of the Republic of Slovakia).

17. We assume that there is no long-lasting dependency between the vote-buyer and the voters. Otherwise, even repeated voting might not prevent vote-buying.

18. Delayed implementation is a common practice. An example has recently taken place in Germany, where the increase of the official retirement age from 65 to 67 will become effective only after the current older generation has retired [see Gersbach and Kleinschmidt (2009)].

19. It suffices that there is a very small level of altruism.

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APPENDIX: PROOFS

A.1. PROOF OF PROPOSITION 1

In the following, we show that there exists a unique subgame-perfect equilibrium in the vote-buying game with constitutional rules TFM, RoA, and BB and the tie-breaking rules TR 1–TR 3 in which the randomly chosen agenda-setter makes no redistribution proposal if the moral costs of vote-selling are sufficiently low.

To prove the result, we proceed in three steps. In the first and second step, we examine the second stage of the voting game, i.e., the subgame that follows if the agenda-setter has made a proposal in the first stage. In the third step, we consider the first stage of the voting game. We use the results of Steps 1 and 2 to identify the subgame-perfect Nash equilibrium of the entire game in Step 3.

Step 1: Consider a proposal where the agenda-setter taxes at most $(n-1)/2$ individuals and uses a part of the tax revenues B_t to subsidize himself ($s_t^{\text{ag}} > 0$) and the remaining part of the tax revenues ($B_t - s_t^{\text{ag}}$) to subsidize untaxed individuals to form a coalition that supports his proposal. In the following, we will use \mathcal{T} ($0 < \mathcal{T} \leq (n-1)/2$) to denote the number of taxed individuals and \mathcal{S} to denote the number of untaxed individuals who receive positive subsidies. Accordingly, the maximal number of untaxed individuals that the agenda-setter can subsidize amounts to $n-1-\mathcal{T}$.

Recall that the taxpayers will form a coalition and bribe the $(n+1)/2 - \mathcal{T}$ least expensive untaxed individuals if condition VB(2) holds. To make vote-buying most expensive and thus least attractive for coalition of taxpayers, we consider proposals where the agenda-setter makes equal subsidies to all individuals he subsidizes. That is, all subsidized individuals receive the same subsidy, which is given by $s_t = (B_t - s_t^{\text{ag}})/\mathcal{S}$.

We now show that the total bribes of the coalition of taxpayers needed to defend the proposal of the agenda-setter are maximal if the agenda-setter makes a proposal with equal subsidies for all $n-1-\mathcal{T}$ untaxed individuals.

If $0 \leq \mathcal{S} \leq (n-3)/2$, then, according to TR 3, the coalition of taxpayers can defeat the proposal of the agenda-setter by paying ϕ to $(n+1)/2 - \mathcal{T}$ nonsubsidized individuals. The total bribes of the coalition of taxpayers, denoted by P , for $\mathcal{S} \leq (n-3)/2$ are then given by

$$P = \left(\frac{n+1}{2} - \mathcal{T} \right) \phi. \quad (\text{A.1})$$

If $(n-3)/2 < \mathcal{S} \leq n-1-\mathcal{T}$, then, according to TR 3, the coalition of taxpayers has to pay $s_t + \phi$ to $\mathcal{S} - (n-3)/2$ subsidized individuals and ϕ to the remaining $n-1-\mathcal{T}-\mathcal{S}$ nonsubsidized individuals to form a least expensive majority against the proposal. The total bribes of the coalition of taxpayers for $\mathcal{S} > (n-3)/2$ are given by

$$\begin{aligned} P &= (n-1-\mathcal{T}-\mathcal{S})\phi + \left(\mathcal{S} - \frac{n-3}{2} \right) (s_t + \phi) \\ &= \left(\frac{n+1}{2} - \mathcal{T} \right) \phi + \left(\mathcal{S} - \frac{n-3}{2} \right) \frac{B_t - s_t^{\text{ag}}}{\mathcal{S}}, \end{aligned} \quad (\text{A.2})$$

where we have used the fact that $s_t = (B_t - s_t^{\text{ag}})/\mathcal{S}$ for each subsidized individual. Comparing (A.1) with (A.2) yields that the total bribes for the coalition of taxpayers

are larger if $(n-3)/2 < \mathcal{S}$. Moreover, analyzing (A.2) yields that the total bribes for the coalition of taxpayers are maximal if $\mathcal{S} = n-1-\mathcal{T}$, as P is strictly increasing in \mathcal{S} .

Step 2: In the second step, we show that, given that the moral costs of vote-selling are sufficiently low, it is always profitable for the coalition of taxpayers to engage in vote-buying, even if the agenda-setter makes a proposal with equal subsidies to all $n-1-\mathcal{T}$ untaxed individuals.

For $\mathcal{S} = n-1-\mathcal{T}$, the total bribes of the coalition of taxpayers are given by

$$P(n-1-\mathcal{T}) = \left(\frac{n+1}{2} - \mathcal{T}\right) \cdot \left(\frac{B_t - s_t^{\text{ag}}}{n-1-\mathcal{T}} + \phi\right). \quad (\text{A.3})$$

Note that initially all individuals are caught in the poverty trap ($\lambda_0^i = 1 \ \forall i \in \Omega$). According to TFM, τ^{sub} is the highest taxation allowed for households in a state of backwardness. Hence, the expected total tax revenue for the agenda-setter is given by

$$B_t = \mathcal{T}\tau^{\text{sub}}. \quad (\text{A.4})$$

We now examine the conditions under which the taxpayers will form a coalition and engage in vote-buying. According to VB(2), taxpayers will form a coalition and buy votes if the gain of vote-buying, which is given by

$$B_t - P(n-1-\mathcal{T}) = \mathcal{T}\tau^{\text{sub}} - \left(\frac{n+1}{2} - \mathcal{T}\right) \cdot \left(\frac{B_t - s_t^{\text{ag}}}{n-1-\mathcal{T}} + \phi\right), \quad (\text{A.5})$$

is weakly positive. The expression in (A.5) is weakly positive if the moral costs of vote-selling, ϕ , are sufficiently small, i.e., if the following condition holds true:

$$\phi \leq \frac{(n-3)\mathcal{T}\tau^{\text{sub}} + (n+1-2\mathcal{T})s_t^{\text{ag}}}{(n+1-2\mathcal{T})(n-1-\mathcal{T})}. \quad (\text{A.6})$$

Note that we have assumed that $n > 3$ and $0 < \mathcal{T} \leq (n-1)/2$. Now suppose that the subsidies for the agenda setter are arbitrarily small but positive. Formally, for $s_t^{\text{ag}} \rightarrow 0$,

$$\frac{(n-3)\mathcal{T}\tau^{\text{sub}} + (n+1-2\mathcal{T})s_t^{\text{ag}}}{(n+1-2\mathcal{T})(n-1-\mathcal{T})}$$

converges to

$$\bar{\phi}(\mathcal{T}) = \frac{(n-3)\mathcal{T}\tau^{\text{sub}}}{(n+1-2\mathcal{T})(n-1-\mathcal{T})}.$$

That is,

$$\bar{\phi}(\mathcal{T}) \leq \frac{(n-3)\mathcal{T}\tau^{\text{sub}} + (n+1-2\mathcal{T})s_t^{\text{ag}}}{(n+1-2\mathcal{T})(n-1-\mathcal{T})}$$

for every $s_t^{\text{ag}} > 0$. So condition (A.6) is fulfilled if $\phi \leq \bar{\phi}(\mathcal{T})$ for every $s_t^{\text{ag}} > 0$. Also note that $\bar{\phi}(\mathcal{T})$ is increasing in \mathcal{T} . That is, for $\mathcal{T} = 1$ and $n > 3$, we obtain

$$\bar{\phi}(1) = \frac{(n-3)\tau^{\text{sub}}}{(n-1)(n-2)} > 0.$$

If $\phi \leq \bar{\phi}(1)$ holds true, then the potential gain from vote-buying for the coalition of taxpayers is positive for every $T \in [1, (n-1)/2]$ and every $s_t^{\text{ag}} > 0$. Thus, it is profitable for the coalition of taxpayers to engage in vote-buying. Hence, the proposal will not be adopted.

Step 3: We now turn to the first stage of the vote-buying game. According to tie-breaking rule TR 1, the agenda-setter will never apply for agenda-setting, because he expects that every proposal with $s_t^{\text{ag}} > 0$ to be rejected with certainty if the moral costs of vote-selling are sufficiently small. Hence, no growth-promoting redistribution occurs in period t , which implies that the human capital in the next period amounts to $\lambda_{t+1}^i = 1$ for all individuals.

Note that the preceding argument holds true for every period t . Hence, the education of a society is not possible in finite time. ■

A.2. PROOF OF PROPOSITION 2

In the following, we show that there exists a subgame-perfect equilibrium in the vote-buying game with constitutional rules TFM, RoA, BB, and RoV and the tie-breaking rules TR 1–TR 3 in which the agenda-setter makes a growth-promoting redistribution proposal, taxpayers do not engage in vote-buying, and the proposal is accepted in the first vote.

To show this result, we have to proceed in three steps. In the first step, we examine the second stage of the voting game. In particular, we derive the condition for the number of voting repetitions where vote-buying is never profitable for the coalition of taxpayers. In the second step, we consider the optimal behavior of the agenda-setter in the first stage of the voting game. In the third step, we show that a democracy with TFM, RoA, BB, and RoV can educate a society.

Step 1: We will now derive the condition for the number of voting repetitions where vote-buying is never profitable for the coalition of taxpayers. The repetitions of votes are indexed by $r \in \{0, 1, \dots, R\}$. Let $B_t(r)$ denote the total expected tax revenue in period t if the voting is repeated r times. Let $s_t^i(r)$ denote the subsidy that individual $i \in \text{NT}$ will receive from the agenda-setter in period t if the proposal is accepted after having been rejected r times before. Correspondingly, let $p_t^i(r)$ denote the payments that individual $i \in \text{NT}$ would receive in period t from the coalition of taxpayers, if he again voted against the proposal that had already been rejected r times before.

In the following, we focus on the proposal where the agenda setter will tax $(n-1)/2$ individuals and pay no subsidies to the untaxed individuals. That is, $s_t^{\text{ag}}(r) = B_t(r)$ and $s_t^i(r) = 0 \forall i \in \text{NT}$ and $\forall r \in \{0, 1, \dots, R\}$. Note that, according to TR 2, all untaxed individuals will vote in favor of this proposal if they are not bribed by the coalition of taxpayers; i.e., the proposal is adopted without vote-buying.

Recall that, initially, all individuals are caught in the poverty trap, $\lambda_0^i = 1 \forall i \in \Omega$. According to TFM, τ^{sub} is the highest taxation allowed for households in a state of backwardness. So the expected tax revenues in the first round, i.e., when the number of repetitions is zero, are given by

$$B_t(0) = \frac{n-1}{2} \tau^{\text{sub}}. \quad (\text{A.7})$$

Recall that we have assumed that

$$\frac{n-1}{2}\tau^{\text{sub}} \geq \bar{s},$$

i.e., $s_t^{\text{ag}}(0) \geq \bar{s}$.

Because the agenda-setter will tax $(n-1)/2$ individuals, it suffices for the coalition of taxpayers to buy only one untaxed individual in order to form a minimal coalition that will vote against the proposal, and to pay this individual

$$p_t(r) = \phi > 0$$

in each vote (see TR 3). In order to win the vote against the proposal R times, the coalition of taxpayers has to pay a total amount of

$$\begin{aligned} \sum_{r=0}^R p_t(r) &= [p_t(0) + p_t(1) + \dots + p_t(R)] \\ &= \phi R. \end{aligned} \tag{A.8}$$

We are now able to derive the number of repetitions of this proposal that will ensure that vote-buying will not be attractive for the coalition of taxpayers. The condition where vote-buying is never profitable for the coalition of taxpayers is given by

$$\sum_{r=0}^R p_t(r) \geq B_t(0).$$

Hence, the number of repetitions R^* where vote-buying will be not profitable for the coalition of taxpayers is implicitly given by

$$\sum_{r=0}^{R^*} p_t(r) = B_t(0). \tag{A.9}$$

We now return to equation (A.9) in light of equations (A.7) and (A.8). We obtain

$$\phi R^* = \frac{n-1}{2}\tau^{\text{sub}}. \tag{A.10}$$

Rearranging equation (A.10) yields

$$R^* = \frac{(n-1)\tau^{\text{sub}}}{2\phi}.$$

Because R^* is a positive real number, we have to use the ceiling function for R^* . The ceiling function is denoted by $\lceil R^* \rceil$, and it denotes the minimal natural number larger than or equal to R^* .

To sum up: If the voting is only repeated $R < \lceil R^* \rceil$ times, then it is profitable for the coalition of taxpayers to buy votes, because $\sum_{r=1}^R p_t(r) < B_t(0)$. However, if $R \geq \lceil R^* \rceil$, then it is optimal for the coalition of taxpayers not to engage in vote-buying, which implies that the proposal will be adopted in the first vote.

Step 2: We now turn to the first stage of the vote-buying game. In this step, we examine the optimal behavior of the agenda-setter in the first stage of the voting game. In Step 1, we have seen that the proposal of the agenda-setter—where he taxes $(n - 1)/2$ uneducated individuals with τ^{sub} to subsidize himself with at least \bar{s} —will be adopted in the first vote, if the voting on this proposal can be repeated $\lceil R^* \rceil$ times. Now we show that no profitable deviations exist for the agenda-setter.

First, it is not profitable for the agenda-setter to make a proposal where more than $(n - 1)/2$ individuals are taxed. Taxation of more than $(n - 1)/2$ individuals would imply that the agenda-setter cannot strictly improve his utility, because such a proposal would never be accepted, as a majority would always vote against it.

Second, it is also not profitable for the agenda-setter to make a proposal where fewer than $(n - 1)/2$ individuals are taxed. Obviously, these proposals would also be accepted. However, taxing fewer than $(n - 1)/2$ individuals would entail a reduction of the subsidies for the agenda-setter. Hence, taxing fewer than $(n - 1)/2$ individuals is not profitable for the agenda-setter either.

Third, it is not profitable for the agenda-setter to make a proposal where he taxes uneducated individuals by more than τ^{sub} . According to TFM, this proposal could be prevented easily by the taxpayers without vote-buying, as the unanimity rule would prevail in this case.

Fourth, it is not profitable for the agenda-setter to make a proposal where he taxes uneducated individuals by less than τ^{sub} . Obviously, this proposal would be accepted by a majority also. However, taxing uneducated individuals with less than τ^{sub} would entail a reduction of the subsidies for the agenda-setter.

Finally, it is not profitable for the agenda-setter to pay positive subsidies to the untaxed individuals. It appears that proposals of this kind would also be accepted, because the subsidizing of untaxed individuals would make vote-buying more costly and therefore less attractive for the coalition of taxpayers. However, subsidizing untaxed individuals would also lead to lower subsidies for the agenda-setter. Hence, subsidizing of untaxed individuals is not profitable for the agenda-setter either.

Step 3: In Steps 1 and 2, we have shown that there exists a subgame-perfect equilibrium in the vote-buying game with constitutional rules TFM, RoA, BB, and RoV and the tie-breaking rules TR 1–TR 3 in which the agenda setter makes a growth-promoting redistribution proposal, taxpayers do not engage in vote-buying, and the proposal is accepted in the first vote. We now show that such a democracy can educate a society.

Because of the rotating agenda-setting rule (RoA), each individual will have the right to set the agenda. RoV ensures that vote-buying will not occur, which implies that each individual will receive the required transfer \bar{s} . The threshold flexible-majority rule, $\text{TFM}[\tau_t^{\text{max}}, \bar{\tau}]$, with $\bar{\tau} = \min\{\tau^{\text{sub}}, \tau^*\}$, ensures both that uneducated households will not fall below the subsistence level and that educated households will not fall back into the poverty trap. Hence, the education achieved in the period of transfer yields a human capital amounting to $\lambda > \lambda^*$ in the next and in the following periods, which implies that the society will be educated in $T < \infty$. ■